Homework 3

Theory of Computation (CSCI 2210)

due: 2023-10-15

Problem 1
Recall our expression language for arithmetic, $(\underline{0}\,, \underline{1}\,, \underline{+}\,, \underline{\times})$ and the two interpretations that we gave for it, one in the natural numbers, $(0\,, 1\,, +\,, \times)$, and the other in the booleans, $(\bot\,, \top\,, \vee\,, \wedge)$.

Give an example of a syntactic equation $\underline{\text{lexp}} = \underline{\text{rexp}}$ that is satisfied in the boolean interpretation but not in the natural numbers interpretation.

Hint: notice that $\mathbb{B} := \{\top, \bot\}$ has only two elements while $\mathbb{N} := \{0, 1, 2, \cdots\}$ has a lot more.

Problem 2
Give regular expressions for each of the following languages over the alphabet $\{0, 1\}$:

(a) words that have odd length,

example: 0 1 0

counterexample: 0 1 0 1

(b) words that begin in 0 and have even length or that begin in 1 and have odd length,

examples: 0 1, 1 0 1

counterexamples: 0 1 1, 1 0

(c) words containing at most two consecutive 1s,

examples: 0 1 0, 1 1 0 1 1

counterexamples: 1 1 1, 0 1 0 1 1 0 1 1 1

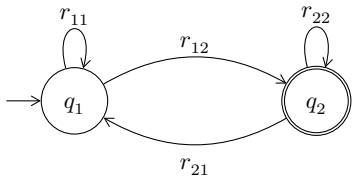
Problem 3
Practical implementations of regular expressions often include additional symbols not included in our definition, such as $-\underline{?}$ for an *optional* regular expression and $-\underline{^+}$ for *at least one iteration* of a regular expression. The reason that we don't need to include these in the formal definition of regular expressions is they are *definable* in terms of the symbols we already have.

Define the symbols $\underline{?}$ and $\underline{^+}$ in terms of the symbols $\{\underline{\emptyset}, \underline{\varepsilon}, \underline{^*}, \underline{+}, \underline{\cdot}\}$ by completing the following:

- $\underline{r?} :=$ some regular expression involving r
- $\underline{r^+} :=$ some regular expression involving r

Problem 4

Apply the recursive reduction algorithm for converting an NFA to a regular expression to the following NFA:



by *first* eliminating vertex q_2 and *then* eliminating vertex q_1, and verify that you obtain the regular expression $(r_{11} + r_{12}\, {r_{22}}^*\, r_{21})^*\, r_{12}\, {r_{22}}^*$ as was claimed in lecture. Please show the state of the graph after each step of the algorithm since I already told you the result.

Problem 5

Use the *pumping lemma* to show that the language of balanced parentheses over the alphabet {"{", "}"} (i.e., the two-element set consisting of opening parenthesis and closing parenthesis) is not regular.

Hint: for the *carefully chosen word* to apply the pumping lemma to, try $\{^p \cdot \}^p$ (i.e., p opening parentheses followed by p closing parentheses) where p is the pumping length for this language.