Homework 2

Theory of Computation (CSCI 2210)

due: 2023-10-01

Problem 1
Draw a diagram for the state transition graph of the following DFA:

input alphabet: $\Sigma := \{0, 1\}$
state set: $\mathrm{Q} := \{q_0, q_1, q_2, q_3, q_4\}$
start state: $q_0 := q_0$
accept state set: $\mathrm{F} := \{q_4\}$

transition function: $\delta :=$

	0	1
q_0	q_1	q_3
q_1	q_1	q_2
q_2	q_4	q_2
q_3	q_3	q_3
q_4	q_4	q_3

and give a brief English description of the language that it decides.

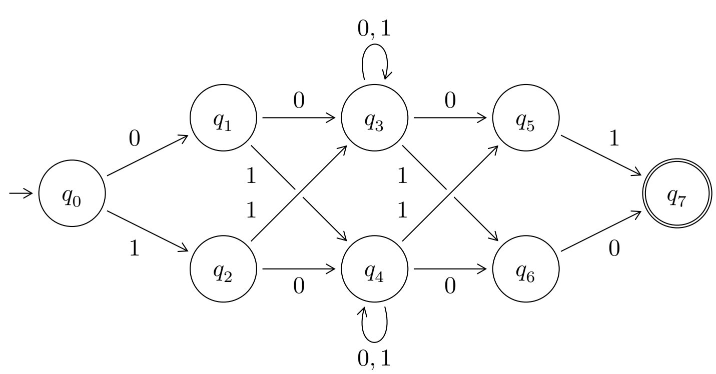
Problem 2
The *intersection* of a pair of languages L_0 and L_1 over the alphabet Σ is the language that contains those words contained in both of them; that is,

$$\overbrace{\mathrm{L}_0 \cap \mathrm{L}_1}^{\text{as languages}} := \underbrace{\mathrm{L}_0 \cap \mathrm{L}_1}_{\text{as subsets of } \Sigma^*}.$$

The intersection of regular languages is regular. Demonstrate this by giving a construction that takes as inputs a pair of DFAs M_0 and M_1 that decide languages L_0 and L_1 respectively, and produces a DFA that decides the language $\mathrm{L}_0 \cap \mathrm{L}_1$.

Hint: You may want to review the construction that we gave in class for the DFA that decides the union language $\mathrm{L}_0 \cup \mathrm{L}_1$.

Problem 3
Consider the NFA represented by the following state transition graph:



(a) What is the length of the shortest word accepted by this NFA?

(b) Give the description of a *run* (i.e. a path in the state transition graph) of this NFA on a word that it accepts.

(c) Give an example of a word at least as long as your answer to part (a) that this NFA does not accept, and explain how we know that there is no accepting run for this word.

(d) Give an English description of the language of this NFA.

Problem 4

Every NFA is equivalent to an ε-NFA (i.e. an NFA that may have ε-transitions) with exactly one accepting state. Demonstrate this by giving a construction that takes as input an arbitrary NFA (with or without ε-transitions) and produces an ε-NFA with exactly one accepting state that decides the same language.

Problem 5

Suppose that NFAs M_0, M_1, and M_2, decide languages L_0, L_1, and L_2, respectively. Draw a schematic diagram for an ε-NFA that decides the language $(\mathrm{L}_0 \mathbin{+\!\!+} \mathrm{L}_1) \cup (\mathrm{L}_2{}^*)$

Problem 6

Apply the *powerset construction* to produce a DFA that decides the same language as is decided by the following NFA: