

Homework 1

Theory of Computation (CSCI 2210)

due: 2023-09-13

Problem 1

For each part give an example of two nonempty finite sets A and B such that:

- (a) $|A \cup B| = |A| + |B|$
- (b) $|A \cup B| < |A| + |B|$
- (c) $|A - B| = |A| - |B|$
- (d) $|A - B| > |A| - |B|$

Problem 2

For each part give an example of a function $f : \mathbb{N} \rightarrow \mathbb{N}$ that is:

- (a) injective but not surjective
- (b) surjective but not injective
- (c) neither injective nor surjective

Problem 3

A function from a set to itself is called an *endomorphism*. Let $\text{Endo}(A)$ be the set of all endomorphisms on the set A; that is, $f \in \text{Endo}(A)$ just in case $f : A \rightarrow A$.

- (a) Show that for any set A the binary operation of *function composition* makes $\text{Endo}(A)$ into a *semigroup*.
Hint: expand the definition of semigroup and recall that two parallel functions are equal just in case they agree on where to send each element of their domain.
- (b) In fact, $\text{Endo}(A)$ is a *monoid*, what is the *neutral element* for function composition?

Problem 4

Explain why if a graph contains a *cycle* then it must contain infinitely many *parallel paths*.

Problem 5

Let A be the set {rock, paper, scissors}.

- (a) Draw a graph representing the relation beats : $A \leftrightarrow A$ in the game *rock-paper-scissors*.
- (b) Which of the following properties does the beats relation possess: reflexivity, transitivity, symmetry?
- (c) Draw a graph representing the converse relation, beats^o.
- (d) Draw a graph representing the composite relation, beats \odot beats.