Practice Quiz

CSCI 2210

2023-11-15

Elaborating Lambda Terms

Elaborate the following λ-term making explicit the implied parentheses and expanding multiple variable bindings into successive single bindings:

$$\lambda x\, y\, z\, .\, x\, z\, (y\, z)$$

Variable Binding

Annotate the binding structure of the following λ-term by drawing an arrow from each bound occurrence of a variable to its binding site.

$$(\lambda x \,.\, (\lambda x \,.\, x)\, x\, ((\lambda x \,.\, x)\, x))\, x$$

Bound Variable Renaming

The *Barendregt convention* on bound variables is to use α-equivalence to rename them such that:

- bound variable names are distinct from free variable names,
- a bound variable name is used at only a single binding site within a term.

This way it is easy to tell whether a variable occurrence is bound, and if so, where.

Apply the Barendregt convention to the term,

$$(\lambda x \,.\, (\lambda x \,.\, x)\, x\, ((\lambda x \,.\, x)\, x))\, x$$

Beta-Reduction of Terms

β-reduce the following λ-terms to their normal forms, if they have them, if not briefly explain why not.

- $(\lambda x\, y \,.\, x)\, (\lambda x \,.\, y)\, x$
- $(\lambda x\, y \,.\, y)\, ((\lambda x \,.\, x\, x)\, (\lambda x \,.\, x\, x))\, z$

Solving Fixed Point Equations

Recall that the Y**-combinator** gives a *fixed point* of any λ-term.
$\mathrm{Y} \;=\; \lambda f \,.\, (\lambda x \,.\, f\,(x\,x))\,(\lambda x \,.\, f\,(x\,x))$.

Here is a recursive definition of the addition function:

$$\mathrm{add} = \lambda m \,.\, \lambda n \,.\, \mathrm{is0}\, m\, n\, (\mathrm{succ}\,(\mathrm{add}\,(\mathrm{pred}\, m)\, n))$$

The pure λ-calculus does not directly support recursion, but it can be implemented using fixed points.

Rewrite the function add as a fixed point of a function F such that,

$$\mathrm{Y\,F}\,2\,3 \downarrow_{\beta} 5$$